MATHEMATICAL LOGIC HOMEWORK 7

Due Monday, April 29.

Problem 1. *Prove that the following sets are* Σ_3^0*:*

(1) $Cofin := \{e \mid \mathbb{N} \setminus W_e \text{ is finite }\}$.
(2) $\{(a,b) \mid W_a \subset^* W_b\}$, *where* $A \subset^* B$ *means that* $A \setminus B$ *is finite.*
(3) $\{(a,b) \mid$ *there is a recursive* C *s.t* $A \subset C \wedge B\ C = \emptyset\}$.

Problem 2. *Classify the following in the arithmetic hierarchy:*

(1) $\{e \mid W_e \subset \{0,1\}\}$.
(2) $\{e \mid W_e \neq \emptyset \wedge W_e \text{ is finite }\}$.

Problem 3. *Prove that there is some number* z*, such that* $W_z = \{n \mid n \geq z\}$.

Problem 4. *Prove that the relation* $\{(e,m) \mid W_e = W_m\}$ *is* Π_2*, but not* Σ_2.

Problem 5. *Let* f *be a total recursive function. Show that* $B := \{e \mid \phi_e = f\}$ *is* Π_2*-complete.*